Coalitional Game Forms with Topological Closure Systems

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This paper contributes to the task of classifying game forms from a structural point of view by studying properties of their concept-or Galois- lattices. In particular, we focus on those coalitional game forms having topological closure systems.

A coalitional game form (CGF) is a triple G =(N; X; E) where N is the player set, X is the outcome set, and E : PN ! PP X is the e®ectivity function. The concept lattice of G is de¯ned as follows:

for any S µ PPN , A µ PPX posit

h$_E$(S) = fA µ X : A 2 E(S) for all S 2 Sg and

i$_E$(A) = fS µ N : A 2 E(S) for all A 2 Ag:

Then, consider

C(G) = f(S; A) 2 PPN £ PPX : S = **i**$_=$(A); and A = **h**$_=$(S)g:

In the language of formal concept analysis an element (S; A) of C(G) is said to be a concept of the context G; with extent D and intent Y (the latter notions are amenable to straightforward dualizations).

The concept lattice of G (sometimes also referred to as its Galois lattice) is L(G) = (C(G); º) where for any $(S_1; A_1); (S_2; A_2)$ 2 C(G)

$(S_1; A_1)$ ¸ $(S_2; A_2)$ i® A_1 ¶ A_2 (which is provably equivalent to S_2 µ S_1), and

$(S_1; A_1)$ ^ $(S_2; A_2)$ = (**i**$_=$(**h**$_=$(S_1 [S_2)); A_1 \ A_2)

$(S_1; A_1)$ _ $(S_2; A_2)$ = (S_1 \ S_2; **h**$_=$(**i**$_=$(A_1 [A_2)):

It is also well-known and easily shown that both (**i**$_=$ ± **h**$_=$) : PPN ! PPN and (**h**$_=$ ± **i**$_=$) : PPX ! PPX are closure operators with respect to set-inclusion (recall that a closure operator K on a preordered set (Y; ¸) is a function K : Y ! Y such that for any y; x 2 Y : K(y) ¸ y; K(y) ¸ K(x) whenever y ¸ x, and K(y) ¸ K(K(y))), and extents and intents of concepts are precisely the closed elements of (**i**$_E$ ± **h**$_E$)and (**h**$_E$ ± **i**$_E$) respectively (i.e.

$(S, A) \in C(G)$ iff $S = \mathbf{i}_E(\mathbf{h}_E(S))$ and $A = \mathbf{h}_E(\mathbf{i}_E(A)))$ and comprise the (Galois) closure systems of CGF G.

A closure operator K on $(PZ, \leq)$ and its corresponding closure system are topological if K also satisfies normality i.e. $K(\emptyset) = \emptyset$ and additivity i.e. $K(C \cup D) = K(C) \cup K(D)$.

This paper addresses the following issue: under what circumstances are the closure operators $K_E = (\mathbf{i}_E \circ \mathbf{h}_E)$ and $K_E^* = (\mathbf{h}_E \circ \mathbf{i}_E)$ topological?

It is shown that the following proposition holds:

Theorem: Let $G = (N, X, E)$ be a monotonic CGF. Then K_E is topological iff G is $\emptyset$-normalized and the monogenic K_E^*-closed sets are meet-irreducible (a dual result obtains for K_E).

It is also shown that CGFs with topological closure systems include additive effectivity functions and simple effectivity functions, but do not reduce to them, and that the resulting topologies are T_0 only if the underlying CGF is 'purified'.

References

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