

# Discontinuous pseudo-games and applications

Jacqueline Morgan

Vincenzo Scalzo

*E-Mail: morgan@unina.it*

*E-Mail: scalzo@unina.it*

*Dipartimento di Matematica e Statistica, Università di Napoli Federico II*

*Via Cinthia, 80126 Napoli, Italy*

In this talk, we present a class of abstract economies, depending on a parameter and with possibly discontinuous payoffs, which guarantees existence, variational stability and parametric well-posedness of its social Nash equilibria. More precisely, for  $m \geq 2$ , let  $X, Y_1, \dots, Y_m$  be  $m + 1$  sequential spaces and, for any  $i \in \{1, \dots, m\}$ , let  $f_i$  be an extended real valued function defined on  $X \times Y$ , where  $Y = \times_{j=1}^m Y_j$ , and  $K_i$  be a set-valued function from  $X \times Y_{-i}$  to  $Y_i$ , where  $Y_{-i} = \times_{j \neq i} Y_j$ . For any  $x \in X$ , let  $\Gamma(x)$  be the *abstract economy* ([5]) or *pseudo-game* ([6]) defined by:

$$\Gamma(x) = \{Y_i, K_i(x, \cdot), f_i(x, \cdot) / i \in \{1, \dots, m\}\}$$

For games with discontinuous payoffs, existence of Nash equilibria has been investigated, among others, in [1], [4] and [9], as well as existence of social Nash equilibria, as introduced in [5], in [7] and [10].

When the data are depending also on an exogenous parameter, sufficient conditions for variational stability (that is the closedness of the set-valued function defined by the social Nash equilibria) have been given in [3] and [7] for possibly non continuous payoffs.

Finally, we recall that in [2] well-posedness (which guarantees the convergence of suitable "approximating sequences" to some social Nash equilibrium) has been introduced and investigated for zero-sum games with non continuous payoffs, extending the notion introduced in optimization by Tykhonov.

Using the classes of *sequentially upper and lower pseudocontinuous functions*, recently introduced by the authors in [8], we present *explicit* sufficient conditions on any data which guarantee existence, variational stability and parametric well-posedness of the social Nash equilibria, together with the connections between games with pseudocontinuous payoffs and others classes of discontinuous games present in literature.

Finally, we consider a finite economy in which the data are depending on an exogenous parameter whose values may correspond to several states of the

world. In this case, for theoretical or numerical reasons, one would like to predict the behavior of competitive allocations (or approximations of those) corresponding to perturbations of the parameter when the utilities of the consumers are discontinuous. Since the parametric well-posedness is satisfied for pseudo-games with pseudocontinuous payoffs, this allows to obtain some "stability" result when the utilities are pseudocontinuous functions.

## References

- [1] BAYE M. R., TIAN G. and ZHOU J., "Characterizations of the existence of equilibria in games with discontinuous and non-quasiconcave payoffs", *Review of Economic Studies*, Vol. 60, pp. 935-948, 1993.
- [2] CAVAZZUTI E. and MORGAN J., "Well-Posed Saddle Point Problems", *Optimization Theory and Algorithms*, Edited by J. B. Hiriart-Urruty, W. Oettli and J. Stoer, Lecture Notes in Pure and Applied Mathematics, M. Dekker, Vol. 86, pp. 61-76, 1983.
- [3] CAVAZZUTI E. and PACCHIAROTTI N., "Convergence of Nash equilibria", *Bollettino U.M.I.*, Vol. 5-B, pp. 247-266, 1986.
- [4] DASGUPTA P. and MASKIN E., "The existence of equilibrium in discontinuous economic games, I: Theory", *Review of Economic Studies*, Vol. 53, pp. 1-26, 1986.
- [5] DEBREU G., "A Social equilibrium existence theorem", *Pro. Nat. Acad. Sci. of USA*, Vol. 38, pp. 886-893, 1952.
- [6] ICHIIISHI T., *Game Theory for Economic Analysis*, Academic Press, New York, 1983.
- [7] LIGNOLA M. B. and MORGAN J., "Existence for optimization problems with equilibrium constraints in reflexive Banach spaces", *Optimization in Economics, Finance and Industry*, Datanova, Milano, Italia, 2002.
- [8] MORGAN J. and SCALZO V., "Pseudocontinuity in optimization and nonzero sun games", *Journal of Optimization Theory and Applications*, Vol. 120 (2), 2004.
- [9] RENY P. J., "On the existence of pure and mixed strategy Nash equilibria in discontinuous games", *Econometrica*, Vol. 67, pp. 1026-1056, 1999.
- [10] TIAN G. and ZHOU J., "Transfer continuities, generalizations of the Weierstrass and maximum theorems: a full characterization", *Journal of Mathematical Economics*, Vol. 24, pp. 281-303, 1995.