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The analysis of poverty, distributive justice and freedom in the Arrow-Debreu set-up

Abstract

In spite of the extensive literature on the Sen's conception of poverty as capability deprivation (cf. Sen 1999, note on this p. 311), this concept is still vague (Quizilbash 2003). It causes in consequence, that the related topics such as the principles of distributive justice and individual freedoms as represented by capability sets are studied in imprecise language. The reason for this seems clear: the capability space has not yet been rigorously specified (Sen 1993, Roemer 1996, Quizilbash 2003).

In this context, the present paper aims to examine these concepts in the unified, formal Arrow-Debreu set-up. The advantage is obvious, the space for action of individuals is uniquely defined as a commodity/prices space of \mathbf{R}^l as well as the states as defined by the Debreu economy (Debreu 1959) are precisely specified what is of importance when interpersonal comparisons are unavoidable. It will be evident that, the approach suggested is not totally devoid of some limitations. Indeed, Sen's functionings as "beings and doings" are reduced to consumption/production plans of economic agents and capability sets to the consumption ones. This simplification, in fact is not so radical as may at first appear, because we confine our analysis to elementary functionings such as (Sen 1992, p. 39) being adequately nourished, being in good health, avoiding escapable morbidity and premature mortality, which can be ensured by carrying out the respective consumption plans in the commodity/price space of \mathbf{R}^l , which a Debreu economy is based on. Moreover, as concerns the poverty seen not as a low level of individual income or resources, but as an inability to convert them into capability to function, it is the inability of elementary functionings to serve essential needs seems to be of the greatest importance because it implies survival deprivation. Thus they enable us to determine the absolute poverty set for each consumer in the form of a set of consumption plans minimizing expenditures on his consumption set as well as comparing the positions of consumers in the states of an economy.

On the other hand, this set-up leads us to the Rawlsian theory of distributive justice (Rawls 1971). Indeed, the location of the individual's absolute poverty set within a budget hyperplane defines the least advantaged members of society, which, together with the postulate to produce the greatest benefit of them, is the core of so-called "Difference Principle" (ibid, p. 90-5). Such an application of interpersonal comparisons enables us to generalize this to the leximin scheme as a relation of l -justice which ranks consumers in a way that the less advantaged in terms of their possibilities of realizing functionings receive a greater precedence. The relation thus defined turns out to be independent of consumer's plans actually achieved and characterizes a Debreu economy as a whole, assuming some elements of the characteristics of a model as fixed. Understood in this way economy is more just, when the situation of the least advantaged consumer improves, if it does not it is up to the next consumer in the worst conditions, and so on. The l - justice relation also satisfies the list of conditions required by the theory of social choice (cf. Arrow 1963, Hammond 1976).

What is more, the independence of the leximin relation of the states of a Debreu economy calls for construction of a space of such models seen as multi-range relational systems with characteristics specifically defined (Malawski 1999). In this framework two problems can be identified and solved. Firstly, a (cumulative) extension of an economy is defined as one in which the l -justice relation is preserved and the respective capability sets offer non-declining ranges of opportunity-freedom. Secondly, the space in question can be interpreted as a state-space of a dynamics. This can be viewed as an evolutionary process of sustaining and enhancing both l -justice relation and substantive individual freedoms that people enjoy.

In this context the paper is divided into five parts. Introduction covers the idea presented here. In the next section, a Debreu economy as a state of a (quasi)-semidynamical system is presented. Section 3 is focused on the study of main concepts of poverty seen as capability deprivation as well as the Rawlsian difference principle and leximin in this framework. Section 4 deals with an extension of a Debreu economy which preserves the relationships already examined. Finally, in Section 5 the static approach used up to now, has been replaced by a dynamic one. The (quasi)-semidynamical Debreu economic system to be constructed guarantees that the relations studied in a static manner are preserved in time-frame.

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