"Procedural" values for cooperative games

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Abstract

According to the beautiful classical probabilistic interpretation of the Shapley value, the value of a player in a cooperative game is his expected marginal contribution to the coalition of his predecessors under random ordering of players, assuming that all possible orderings are equiprobable. We study values which obtain under the same assumption on orders but without the requirement that a player always retains his entire marginal contribution. Instead, we allow for any possible "procedure" of dividing marginal contributions among the members of the coalition, requiring only that nothing is left for the successors.

It is obvious that every procedural value is efficient and linear, and that it is symmetric if the underlying procedure is "anonymous". It is also easy to show that it depends only on the share of the marginal contribution which the contributing player can retain, and not on the way in which the rest is divided among other coalition members. Therefore, sharing rules (and thus procedural values) are defined by sequences of coefficients $(s_{m,n})_{m,n \in N\,,\, m \leq n}$, where $s_{m,n} \in [0,1]$ is the share of a player who arrives as m-th in an n-person game. A value is size-independent when the coefficients $s_{m,n}$ do not depend on n. It turns out that various well-kmown values, including the "egalitarian solution" and the Nowak and Radzik (*IJGT* 1994) "solidarity value", result from simple size-independent sharing procedures. Many other values, e.g. the "equal surplus solution" (Driessen and Funaki, *OR Spektrum* 1991) also obtain as procedural values, but under size-dependent sharing procedures.

For any given game, the (closed and convex) set of all its procedural values is also of interest. A preliminary investigation of its relation with the core and the Harsámyi set is conducted.