

# Core Extensions: Structure, Properties, Axiomatizations

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## Abstract

The concept of the core has been extensively studied since its appearance fifty years ago. Among the classical results of the theory of the core are existence theorems characterizing the games for which the core is nonempty, established for TU games in the sixties and for NTU games in later years.

The idea of extending the solution concept of the core to games for which the core is empty has been around also for many years, but a theory of core extensions seems however to be of more recent origin. The most straightforward and appealing idea, formulated and investigated by Gomez (2003) in the context of TU games, is to multiply the worth of the grand coalition by as little as possible in order to obtain a nonempty core of the resulting game, and then contracting the resulting core payoffs so as to obtain feasible payoffs in the original game. This core extension has some appealing properties, one of which surprisingly connects to the theory of two-person zero-sum games.

The idea of blowing up the feasible set of the grand coalition so as to obtain a game with nonempty core is easily carried over to the class of NTU games, where again it yields an extension of the core defined on all NTU games satisfying weak assumptions of well-behavedness. However, although the operations on games performed are the same, some of the properties, in particular those relating core extension to values of zero-sum games, carry over only in much less elegant form.

On the other hand, the context of NTU games seems to be a more natural one for investigating possible axiomatizations of the extended core. Indeed, there are axiomatizations of the core of NTU games which with few add-ons can be adapted to the situation, thus providing an axiomatic characterization of core extensions. This system of axioms differs slightly from the usual systems characterizing the core, relying on induction through reduced-game and converse-reduced-game properties, using instead a construction of a derived game with the same player set.

The reason why NTU games provide a simpler framework for an axiomatization of extended cores is that some of the intuitively appealing constructions on games entering into the axiomatization will produce an NTU game even if one starts with a TU game. Therefore, a genuine axiomatization of TU extended cores will have to use another approach. Indeed, it may be useful to start with a somewhat weaker concept, here called the aspiration core, for which feasibility is not required, thus relating to previous considerations of aspiration sets (Bennett (1983), Moldovanu and Winter (1994)). However, even so it seems that axiomatizations must involve games which are not TU games.