

# KKM TYPE THEOREMS FOR PRIMOIDS

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## Abstract

Knaster-Kuratowski-Mazurkiewicz type theorems on covering of the geometric simplex are presented [?]. Proofs are based on Sperner lemma type theorems for primoids and pseudomanifolds [?]. Applications of these theorems to the fair division problems are given.

By  $N, R$  we denote the set of natural numbers and reals respectively. Let  $n \in N$  and  $V$  be a finite set of the cardinality at least  $n + 1$ .  $\mathbf{P}(V)$  is the family of all subsets of  $V$  and  $\mathbf{P}_n(V)$  is the family of all subsets of  $V$  of the cardinality  $n + 1$ . An element of  $\mathbf{P}_n(V)$  is called  $n$ -simplex over the set  $V$ .

An  $n$ -dimensional pseudomanifold (or  $n$ -pseudomanifold) is a finite family  $\mathbf{C}$  of  $n$ -simplexes over the set  $V$  having the following property: every  $(n - 1)$ -simplex over  $V$  is a subset of at most two  $n$ -simplexes of  $\mathbf{C}$ .

Let  $U$  be a finite set of the cardinality at least  $n + 1$ . An  $n$ -primoid  $\mathbf{L}_n^U$  over  $U$  is a nonempty family of  $n$ -simplexes over  $U$  fulfilling the following condition: for every  $n$ -simplex  $T \in \mathbf{L}_n^U$  and for any  $u \in U$  there exists exactly one  $u' \in T$  such that an  $n$ -simplex  $T \setminus \{u'\} \cup \{u\} \in \mathbf{L}_n^U$ .

## References

- [1] A. Idzik, K. Junosza-Szaniawski, Combinatorial lemmas for nonoriented pseudomanifolds, *Topological Methods in Nonlinear Analysis* **22** (2003), 387-398.
- [2] B. Knaster, C. Kuratowski, S. Mazurkiewicz, Ein Beweis des Fixpunktsatzes für  $n$ -dimensionale Simplexe, *Fundamenta Mathematicae* **14** (1964), 132-137.

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