The egalitarian solution and the egalitarian split off set for cooperative games

Rodica Branzei
Faculty of Computer Science
"Alexandru Ioan Cuza" University, Iasi, Romania
and
CentER and Department of Econometrics and Operations Research
Tilburg University, The Netherlands

Dinko Dimitrov
CentER and Department of Econometrics and Operations Research
Tilburg University, The Netherlands

Stef Tijs
CentER and Department of Econometrics and Operations Research
Tilburg University, The Netherlands
and
Department of Mathematics
University of Genoa, Italy

A cooperative (reward/cost saving) game with transferable utility (cooperative TU-game) $\langle N, v\rangle$ consists of a nonempty finite set of players $N = \{1, \ldots, n\}$ and a map $v : 2^N \to \mathbb{R}$ with $v(\emptyset) = 0$, where for each $S \in 2^N$, $v(S)$ is to be considered as the reward/cost saving, obtained by coalition S when its members cooperate. A game $\langle N, v\rangle$ is called convex if for each $S, T \in 2^N$, $v(S \cup T) + v(S \cap T) \geq v(S) + v(T)$.

The concept of egalitarianism has generated several solution concepts on the set of cooperative TU-games: the constrained egalitarian solution (Dutta and Ray (1989)), the Lorenz solution (Hougaard et al. (2001)), the Lorenz stable set and the egalitarian core (Arin and Inarra (2001)). The constrained egalitarian solution behaves nicely on the class of convex games: it assigns to each convex game a unique point in the core and it Lorenz dominates each other core allocation. It turns out that all the other egalitarian solutions mentioned above coincide for convex games with the constrained egalitarian solution. On this class of cooperative TU-games alternative axiomatic characterizations of the constrained egalitarian solution are provided by Dutta (1990), Hokari (2000), Klijn et al. (2000). This solution for a convex game can be obtained using the algorithm proposed by Dutta and Ray (1989) or the formula suggested by Hokari (2000). Another solution concept related to the norm of equity is the equal division core proposed by Selten (1972). Selten introduces it in order to explain outcomes in experimental cooperative TU-games and notes that in 76 % of 207 experimental games the outcomes have a "strong tendency to be in the equal division core". The core of a cooperative TU-game is included in the equal division core of that game. Axiomatic characterizations of the equal division core on two classes of cooperative TU-games are provided by Bhattacharya (2004).

The main purpose of this paper is to introduce a new egalitarianism-based solution concept for cooperative TU-games, which we call the equal split off set (ESOS). The ESOS is inspired by the work of Dutta and Ray (1989). They start with arbitrary cooperative TU-games, consider the Lorenz domination, etc., and come to an algorithm for obtaining the egalitarian solution for convex games, which boils down to successive (weak) split off. The notion of average worth of a nonempty coalition S with respect to the characteristic function v of the game, denoted by $a(S, v)$ and referred also to as per capita value, plays here a central role. Recall that $a\left(S, v\right) := v(S)/\left|S\right|$, where $|S|$ stands for the number of players in the coalition S, $S \in 2^N \setminus \{\emptyset\}$.

In our work we consider the Dutta-Ray's successive split off procedure for arbitrary cooperative TU-games which can result now in several divisions of $v(N)$ that are elements of the egalitarian split off set. It turns out that the egalitarian split off set of a convex TU-game coincides with the Dutta-Ray egalitarian solution of that game. We study properties of the egalitarian split off set and search for conditions under which interesting relations with existing egalitarianism-based solution concepts can be provided. Also extensions of egalitarian solutions to fuzzy cooperative games will be discussed (cf. Branzei et al. (2004)).

References

[1] Arin, J., Inarra, E., 2001. Egalitarian solutions in the core, International Journal of Game Theory 30, 187-193.

[2] Bhattacharya, A., 2004. On the equal division core, Social Choice and Welfare 22, 391-399.

[3] Branzei, R., Dimitrov, D., Tijs, S, 2004. Egalitarianism in convex fuzzy games, Mathematical Social Sciences 47, 313-325.

[4] Dutta, B., 1990. The egalitarian solution and reduced game properties in convex games, International Journal of Game Theory 19, 153-169.

[5] Dutta, B., Ray, D., 1989. A concept of egalitarianism under participation constraints, Econometrica 57, 403-422.

[6] Hokari, T., 2000. Population monotonic solutions for convex games, International Journal of Game Theory 29, 327-338.

[7] Hougaard, J., Peleg, B., Thorlund-Petersen, L., 2001. On the set of Lorenz-maximal imputations in the core of a balanced game, International Journal of Game Theory 30, 147-165.

[8] Klijn, F., Slikker, M., Tijs, S., Zarzuelo, J., 2000. The egalitarian solution for convex games: some characterizations, Mathematical Social Sciences 40, 111-121.

[9] Selten, R., 1972. Equal share analysis of characteristic function experiments. In: Sauermann, H., (Ed.), Contributions to Experimentation in Economics. Vol. 3, Mohr, Tübingen, 130-165.