Modelling stock exchange by dynamic games with a continuum of players of finitely many types – extended abstract

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This paper contains a game-theoretic model describing the behaviour of investors at a stock exchange. It is a discrete time dynamic game with continuum of players constituting finitely many types.

The stock exchange, starting form a place where buyers and sellers could face each other and even negotiate prices, evolved to a virtual place in which anonymous masses of investors buy or sell at prices dictated by the equilibrating mechanism. During this process of evolution, as the anonymity increased, various models predicting future prices vere developed: fundamental analysis, technical analysis, various econometric models, CAPM or even... toss of a coin.

The model presented is developed to reflect the actual market microstructure. Therefore it's prognostic properties can be better than that of models presented in books on finance or mathematics of finance.

The players constitute a continuum, divided into a finite number of types. The definition of type involves, among others, planning horizon, external flow of money which can be invested, formation of expectations about future prices, which, briefly, divides the investors into the following groups: fundamentalists, chartist, users of various econometric models, economists using CAPM, and stochastic players. Even audience of financial guru can be included.

A continuum of players is used in order to model a "mature" stock exchange: there are many agents, each of them insignificant. Each single player is conscious that his order cannot affect prices and this reflects real situations. On the other hand prices are effect of agents' orders.

Actual formation of prices is, as in the real life, fully deterministic: prices are determined by orders and equilibrating mechanism of the stock exchange. The mechanism presented in this paper is the single-price auction system used at Warsaw Stock Exchange, defined by lexicographic order with first criterion: maximize volume, second: minimize disequilibrium, third: be as close as possible to the previous price (such a mechanism is used by most stock exchanges).

Depending on sizes of types, even very abstract beliefs can become self-verifying at

least to some extent. In the paper there are examples of such self-veryfying beliefs: some of fundamental nature, technical signals of changes of trends and an absolutely abstract formation of cat. This formation has not existed by now and empirical data does not suggest such a formation. However, it is explained in a quasi-psychological way which is frequently used by authors of textbooks on technical analysis. Moreover, this formation, if it was popularised among investors, would become self-veryfying. This "cat" is an example of self-veryfying character of techniques of foreseeing future prices.

On the other hand, no precise theory describing formation of prices, besides some very strict fundamentalistic schedule (either constant or rapidly increasing or decreasing), can work if it is known and used by all players. Technical analysis, although it is one of the most popular tools used by investors, can have good properties, since it is imprecise: e.g. it is easy to recognize formations only *ex post*.

The continuum of players, although apparently seems a complication, makes the problem easier to solve, since it allows for partial decoupling of the Bellman system of equations for players' optimization problem or even allows to decompose a dynamic game into a sequence of static games (see e.g. [2], [4], [1], [3] or [6]).

The theoretical results are illustrated by computer simulations of behaviour of stock exchange for various parameters of the game (interest rate on bonds, past data series, number of types of players, their measures, and characteristics of these types: formation of expectations, flows of external money etc.).

This paper is a continuation of [3]; it is based on [5] and further examination including computer simulations.

References

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