A Dynamic Spatial Coalition Formation Theory

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So far, it has always been assumed that players in spatial coalition models have a fixed, static position. This article challenges this idea by stating that the formation of a coalition influences the position of players, besides the assumption that the position of players influences the formation of a coalition. This way, we are able to explain how and why players change their position during the coalition formation process. In other words, we assume that the position of a player is not static, but dynamic. The model we have developed is placed in the tradition of spatial modeling and coalition formation theories.

Spatial modeling finds its origins in the work of Anthony Downs (1957) and Duncan Black (1958). By choosing a certain position players want to maximize their utility. Further research has evolved the theory to multi-dimensional modelling. See also the work of Enelow and Hinich (1984) and Hinich and Munger (1984) for a further advancement of the theory of spatial modelling.

Based on these spatial tools coalition formation theories have been built. Coalition formation theories are a branch of game theory and forecast which coalitions will be formed. The theories assume - in general terms - that players will cooperate with "their closest neighbour" (De Vries 1999). That means that given the positions of the players in a multi-dimensional space players that are close to each other will form a coalition.

Spatial coalition formation theories have up till now assumed that players have a static position. At the beginning of a game (and a coalition formation process) the position of the players is determined and will remain the same for the whole game. The theory has not yet been able to explain why and how players change their position during the game.

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The model we have developed is able to explain changes in position of players during a game of coalition formation. We will study these dynamic adjustments by analyzing how the coalition formation process and position of the players influence each other.

The basic ingredients of the model are a group of players with for each individual player a weight w_i, a position x_i in a multi-dimensional Euclidean space R^m, and a *latitude* M_i to restrict their maneuvering space. The players need to reach a quota q to be winning, and since $w_i < q$ each has an incentive to cooperate. A player can change its position during a game, but will never go beyond its latitude. A latitude is a convex and bounded circle (or (hyper)sphere in R^{2+}) and is the set of all potential positions a player wants to take. We assume that the size of a player influences the size of the latitude: the radius of the circle or (hyper-)sphere. The bigger a player is, the smaller his latitude will be, and vice versa. We assume players know each others positions, but lack information on the exact position of the latitude.

To form coalitions players need to find a common position for the coalition. Hence, they have to move their position to reach each other. But since no player will leave its latitude coalitions will only form between players whose latitudes overlap: $M_i \cap M_j \neq \emptyset$.

The negotiating process is always between two players. If they reach an agreement after discussing all dimensions and changing their positions step by step, they will agree on a position x_S in $M_i \cap M_j$. The model explains what exactly will be this position by taking the weights of the players into account. If these two players reach the quota the game stops. If they lack enough weight they need to find an extra partner. Because both players have the same position now, they will together form a new player S. Negotiations will now take place between S and a new player. The latitude of S is the circle around M_i and M_j. The same procedure applies again, until a winning coalition has been reached. If two players cannot find an agreement (i.e. their latitudes do not overlap) a new stage starts with one or two new players.

The idea of the model has been presented in figure 1. Here a space with players and their latitudes is shown. It is clear that some players have an overlap and can cooperate and that some will never work together. The model also emphasizes the dynamics that lead to a coalition. Instead of predicting an outcome right away, the model shows the path that leads to the solution.

The model can be applied in several empirical domains. In e.g. the political science it would mean an enrichment of the analysis of positions of political parties. So far, the positions of parties have only been explained in terms of vote maximizing behaviour (see e.g. Downs (1957), Enelow and Hinich (1984), Hinich and Munger (1984), and Budge (1994)). Besides political coalitions one can also think of strategic alliances between firms. In this field, game theory and (spatial) coalition formation theory have hardly

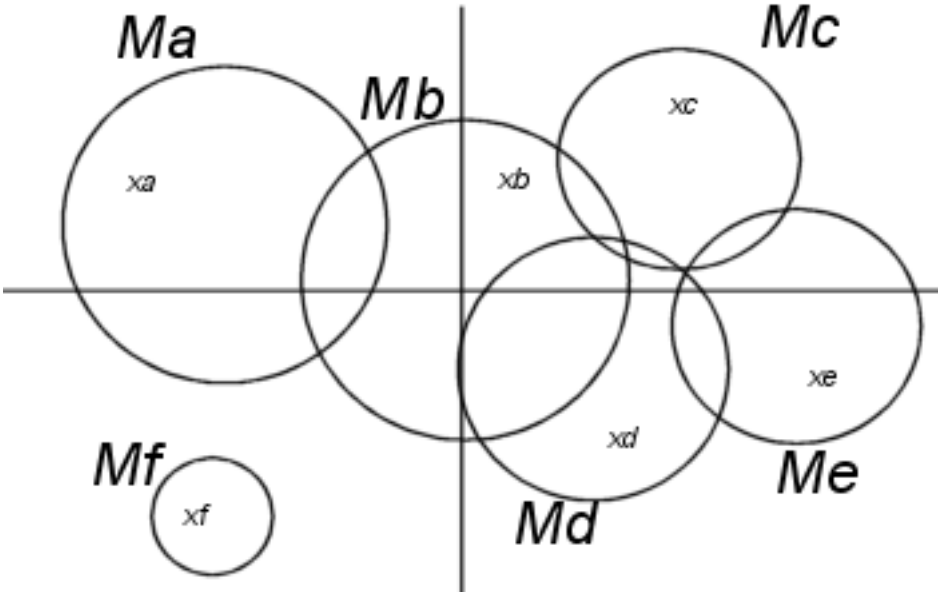

Figure 1: Example of a space with latitudes

been used. This model could really add a new way of looking at alliances.

In conclusion, the model shows how and why players change their position during a coalition formation process. Their position not only shapes the formation of the coalition, but the coalition formation also influences their position. By this model, new insights have been acquired that show this interdependent relation of position and coalition formation.

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