A New Insight for Three Bargaining Solutions in Convex Problems

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Abstract

We provide a new proof of the axiomatic characterization of the Nash solution, and new axiomatic characterizations of the Kalai-Smorodinsky and the egalitarian solutions in the classical domain of bargaining problems. The new characaterizations of the two solutions are based upon the types of the independence axioms, and so that the paper gives us a new insight on the three solutions in terms of rational choice performance of bargaining solutions.

1 Introduction

This paper discusses axiomatic characterizations of bargaining solutions in compact, convex, and comprehensive bargaining problems. Although there have been plenty of papers which discussed this issue, we will add a few new points in the literature of this issue. First, we will provide a new proof of the characterization of the Nash solution, which might be simpler than the standard proof, we hope.

Second, we will provide a new characterization for each of the Kalai-Smorodinsky and the egalitarian solutions by means of the independence types axioms. We will introduce a new axiom, **Expansion Independence**, which is an independence requirement for the case that a bargaining problem expands in a particular way. Then, we will show that for characterizing the egalitarian solution, the standard monotonicity axiom can be replaced by this new axiom and the contraction independence axiom.

Third, we will also introduce two weaker variants of the independence axioms, one of which is a weaker variant of the contraction independence axiom, and the other of which is of the expansion independence axiom. Then, we will show that for characterizing the Kalai-Smorodinsky solution, the weak monotonicity axiom can be replaced by these two variants of the independence axioms. Thus, in conclusion, we will provide a new insight for the above representative bargaining solutions in terms of rationalizability of choice functions.

Our results on the characterizations of the three solutions are summarized in the following table.

Table 1*aroundhere.*

By **Table 1**, while the Nash solution meets only the contraction independence axiom, the egalitarian solution meets both of the contraction and the expansion independence axioms, and the Kalai-Smorodinsky solution meets the weaker variants of the two independence axioms.

Moreover, within the class of (weakly) efficient and symmetric bargaining solutions, the scale invariance axiom is in a trade-off with the expansion independence axiom by Theorems 1 and 2; the contraction independence axiom is in a trade-off with the weak expansion independence axiom by Theorems 1 and 3; and also both the contraction and the expansion independence axioms together are in a trade-off with the scale invariance axiom by Theorems 2 and 3.

It is worth noting that the egalitarian solution has the strongest property among the three solutions in terms of rational choice, since both the contraction and the expansion independence axioms are considered as the necessary conditions for the rationalizability of choice functions. It is also an interesting news that the Kalai-Smorodinsky solution has some constrained property of rational choice; within the class of the problems with the same ideal point, this solution can do a better performance than the Nash solution in terms of the rational choice, since, even within this class of the problems, the Nash solution does not behave as a rational choice function for the expansion of opportunity sets. This gives us a new insight on the rational choice property of the Nash and the Kalai-Smorodinsky solutions, which the previous literature does not, since the Kalai-Smorodinsky solution was considered as having a less rational choice property than the Nash solution.