Correlated Equilibria in Markov Stopping Games. The Main Characterizations

David M. Ramsey*and Krzysztof Szajowski†

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Extended Abstract

1 Introduction

This research is devoted to two-person games based on bilateral stopping of a Markov process. Various models of stopping games, beginning with the Dynkin game (see Dynkin (1969)), have been considered. A review of results can be found in Sakaguchi (1995) and Nowak and Szajowski (1998). A non-zero sum game of this type may not have a unique Nash equilibrium, if the players are only able to choose their strategies from the set of mixed (randomized) stopping rules. Hence, communication between the players would be useful in deciding which equilibrium should be played. Using the idea of correlated strategies introduced by Aumann (1974), the set of possible strategies is extended to the set of correlated stopping times and the actions undertaken by the players are correlated.

Little research has been carried out on the role of communication between players in stopping games. Solan (2001) and Solan and Vieille (2002) consider correlated equilibria in general dynamic games. The form of correlation is not unique. The approach applied here is based on a generalization of randomized stopping times. Various additional criteria used by the players to correlate their actions restrict the set of possible solutions. These criteria are based on those used by Greenwald and Hall (2003), which resemble ideas of solutions of cooperative games presented by Thomson (1994).

Numerical considerations concerning the construction of correlated equilibria in Markov stopping games are presented by Ramsey (see Ramsey and Szajowski (2004)), where examples related to the generalized best choice problem are given.

2 Correlated equilibria in stopping games

Aumann (1974) introduced a correlation scheme in randomized strategies for nonzero-sum games extending the concept of Nash equilibrium. Using this approach some process of preplay communication is needed to realize such a strategy. Aumann's approach has been extended in various manners (eg see Forges (1986); Gerard-Varet and Moulin (1978); Moulin (1986); Nowak (1993); Tolwinski et al. (1986)). The process of adapting correlated equilibria to stopping games starts from the idea of correlated stopping times.

Definition 1 *A random sequence* $\hat{q} = \{(q_n^1, q_n^2, q_n^3)\}$ *such that, for each* n,

(i) q_n^i *is adapted to* $\mathcal{F}_n$ *for* $i = 1, 2, 3$;

(ii) $0 \le q_n^1 \le q_n^2 \le q_n^3 \le 1$ *a.s.*

is called a correlated stopping strategy. The set of all such sequences will be denoted by $\hat{\mathcal{Q}}^N$.

*Instytut Matematyki, Politechniki Wrocławskiej, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, D.Ramsey@im.pwr.wroc.p

†Instytut Matematyki, Politechniki Wrocławskiej, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, K.Szajowski@im.pwr.wroc.pl

Let $A_1, A_2, \ldots, A_N$ be a sequence of i.i.d. r.v. with uniform distribution on $[0,1]$ and independent of the Markov process $(X_n, \mathcal{F}_n, \mathbf{P}_x)_{n=0}^N$. Denote $\vec{q}_n = (q_n^1, q_n^2, q_n^3)$. Correlated stopping times are pairs $(\lambda^1(\hat{q}), \lambda^2(\hat{q}))$ of Markov stopping times with respect to the σ-fields $\mathcal{H}_n = \sigma\{\mathcal{F}_n,\ \{A_1, A_2, ..., A_n\}\}$ defined by the strategy $\hat{q} = (\vec{q}_n) \in \hat{\mathcal{Q}}^N$ as follows:

$$\lambda^1(\hat{q}) = \inf\{0 \le n \le N : A_n \le q_n^2\} \tag{1}$$

and

$$\lambda^2(\hat{q}) = \inf\{0 \le n \le N : A_n \le q_n^1 \text{ or } q_n^2 < A_n \le q_n^3\}. \tag{2}$$

The strategy $\hat{q}$ will be called the correlation profile and it defines the pair of stopping times $(\lambda^1(\hat{q}), \lambda^2(\hat{q}))$.

In intuitive terms, the vector $\vec{q}_n = (q_n^1, q_n^2, q_n^3)$ defines the joint distribution of the actions taken by the players at moment n: with probability q_n^1 both players choose the action "stop", with probability $q_n^2 - q_n^1$ Player 1 stops and Player 2 chooses the action "continue", with probability $q_n^3 - q_n^2$ Player 1 continues and Player 2 stops and with probability $1 - q_n^3$ both players continue. A correlated strategy $\hat{q}$ is assumed to be defined by preplay communication (either before the start of the game or before each decision) between the players with the possible aid of an "external judge". If communication only takes place before the game commences, then such a correlation is said to be a stationary correlation device. If communication may occur at each decision point, then such a correlation is said to be an extensive (autonomous) correlation device (see Solan and Vieille (2002)). In general, we consider extensive correlation devices. The form of the correlation strategy is known to both players.

If one player carries out the actions suggested by a lottery activated by the external judge and the other player departs from the suggested action a formal construction of the possible strategies and the calculation of the expected gains should be done.

Let $\hat{p} = (p_1, p_2, \ldots, p_N)$ be a sequence of random points in the unit interval. If Player i departs from the correlation profile $\hat{q}$, then the strategy of the other player is based on the marginal correlated profile $\hat{q}_{-i}$ and the strategy of Player i is defined by $\hat{p}_i = \hat{p}$. Denote $\tau^i((\hat{p}_i, \hat{q}_{-i})) = \tau^i(\hat{p}_i) = \inf\{0 \le n \le N : A_n^{'} \le p_n\}$, where $(A_n^{'})_{n=1}^N$ is a sequence of i.i.d. r.v. with uniform distribution on $[0,1]$, independent of $(A_n)_{n=1}^N$ and independent of the Markov process $(X_n, \mathcal{F}_n, \mathbf{P}_x)_{n=0}^N$. Denote $\bar{G}_i(\hat{q}) = G_i(\lambda^1(\hat{q}) \wedge \lambda^2(\hat{q}), X_{\lambda^1(\hat{q}) \wedge \lambda^2(\hat{q})})$ and $\bar{G}_i((\hat{p}_i, \hat{q}_{-i})) = G_i(\tau^i(\hat{p}_i) \wedge \lambda^{-i}(\hat{q}_{-i}), X_{\tau^i(\hat{p}_i) \wedge \lambda^{-i}(\hat{q}_{-i})})$. The expected payoffs are defined as $\hat{G}_i(x, \hat{q}) = \mathbf{E}_x \bar{G}_i(\hat{q})$ and $\hat{G}_i(x, (\hat{p}_i, \hat{q}_{-i})) = \mathbf{E}_x \bar{G}_i((\hat{p}_i, \hat{q}_{-i}))$, respectively.

Definition 2 *A correlated stopping strategy $\hat{q}^* \in \hat{\mathcal{Q}}^N$ is called a correlated equilibrium point of $\mathcal{G}_m$, if*

$$\hat{G}_i(x, \hat{q}^*) \ge \hat{G}_i(x, (\hat{p}_i, \hat{q}_{-i}^*)) \tag{3}$$

for every $x \in \mathbb{E}$, $\hat{d}$ and $i = 1, 2$.

3 Selection of a Correlated Equilibrium

Since the set of Nash equilibria is a subset of the set of correlated equilibria, it is clear that whenever the problem of the selection of a Nash equilibria exists, the problem of the selection of a correlated equilibrium also exists. However, the notion of correlated equilibrium assumes that communication takes place. Such communication can be used to define the criteria used by players to select a correlated equilibrium.

Definition 3 *Let us formulate four different selection criteria for correlated equilibria in a stopping game.*

1. *A utilitarian correlated equilibrium $\hat{q}_U^*$ is an equilibrium which at each stage $n = 0, 1, \ldots, N$ maximizes the sum of the values of the game to the players.*

2. *An egalitarian correlated equilibrium $\hat{q}_E^*$ is an equilibrium which at each stage $n = 0, 1, \ldots, N$ maximizes the minimum value of the players rewards.*

3. *A republican correlated equilibrium $\hat{q}_R^*$ is an equilibrium which at each stage $n = 0, 1, \dots, N$ maximizes the maximum value of the players rewards.*

4. *A libertarian i correlated equilibrium $\hat{q}_L^*$ is an equilibrium which at each stage $n = 0, 1, \dots, N$ maximizes the value of the game to Player i.*

Theorem 1 *The set of correlated equilibrium points satisfying any one of the given criteria above is not empty.*

Considerations related to the construction of these types of correlated equilibria in Markov stopping games are presented in Ramsey and Szajowski (2004).

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