Quasi Strong Perfect Equilibrium in the Repeated Cournot Model

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Under our point of view, most of the equilibrium concepts that appear in the literature of infinitely repeated games have a serious drawback: they do not consider the possibility of a group of players forming a coalition to deviate. Subgame perfect equilibrium strategies are defined to avoid single player deviations. Deviations of two or more players are always ignored. We now want to argue that group deviations can not be ignored, and that any deviation of any coalition (other than the grand one) has to be punished by the complementary coalition.

Consider the following example: Let G be a Cournot supergame with five players, lineal demand function given by $p = 100 - z$ (if $z < 100$ and 0 otherwise) and lineal cost function with marginal cost $c = 40$. Let σ_N be the simple strategy profile defined by $S^0 = \{(6,6,6,6,6);(6,6,6,6,6);...\}$; $S^1 = \{(0,10,10,10,10);(6,6,6,6,6);...\}$ and $S^i = S^{1(i/1)}$ $(i = 2,...,10)$. $S^{1(i/1)}$ is identical to S^1 except that the roles of player 1 and player i are interchanged. It is easy to check that σ_N is subgame perfect equilibrium whenever $\delta > 0.8$. Note that σ_N sustains the monopoly payoff $\Pi^m = 180$ which is higher than the Cournot profits $\Pi^c = 100$. Assuming that the rest of the players continue with σ_N, players 1 and 2 can deviate from collusion using the strategy $\sigma_{\{1,2\}}$ given by $S^0 = \{(10.5,10.5);(10.5,10.5);(10.5,10.5);...\}$, and $S^1 = S^2 = \{(14,14);(14,14);(14,14)...\}$. Note that $q^*_{\{1,2\}}(6,6,6)$ (i.e. the quantity that

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maximizes $\Pi^1 + \Pi^2$ given that players 3,4 and 5 continue in the cooperative path) is $21 = 10.5+10.5$, and $q_1^*(14, 6, 6, 6) = 14$, so $\sigma_{\{1,2\}}$ is a trigger strategy that allows players 1, 2 to sustain the payoff 220.5¿180 assuming that the rest of the players continue fix in 6. With simple computations we obtain that neither player 1 nor player 2 will deviate from $\sigma_{\{1,2\}}$ whenever $\delta > 0.529$. In any game with $n > 2$ this kind of trigger strategy could always be used to sustain a two players deviation if there is no reaction on the rest of the players. Of course this is not the only two-player strategy that can be used in this example, but this trigger strategy always works and this means that, in the general case, two players deviations have to be punished because, otherwise, deviations will be carried out in a way that no subcoalition will deviate further.

One of the main results of this paper is proving that in the Cournot supergame with any number of players it is possible to sustain the symmetric monopoly outcome by means of a variety of strategies which satisfy that no coalition (other than the grand one) will deviate in any subgame (provided that the discount factor is close enough to 1).

A straightforward conclusion of this result is that, at least in the symmetric Cournot model, any coalition D (other than the grand one) which has the possibility of improving the payoffs of all of its members with a deviation, has also different strategies to sustain this deviation in a way that no subcoalition will deviate further. This is what allow us to conclude that any deviation of any coalition has to be punished by the complementary coalition.

To avoid the coalitional deviations we introduce the *coalitional simple strategy profiles* which generalize the simple strategy profiles defined by Abreu in (1989). A coalitional simple strategy profile consists of one cooperative path and one punishment path for each non empty coalition (other than the grand one).

These strategies are defined as follows: Start the cooperative path and continue there if no player deviates. If, after any history, any coalition deviates start the punishment phase of this coalition. Only deviations of all the players are ignored.

The equilibrium concept used through the paper is the *Quasi Strong Perfect Equilibrium* (QSPE in brief). An equilibrium is QSPE if no coalition other than the grand one, taking the actions of its complement as given, can deviate in a way that benefits all of its members. We will explain in Section 3 why we can not use the Strong Perfect Equilibrium of Rubinstein (1980).

To obtain all the important results of this paper we have to introduce a second equilibrium concept which is even more strong than QSPE itself. We call it *Quasi Even More Strong Perfect Equilibrium* (QEMSPE). An equilibrium is QEMSPE if no coalition other than the grand one, taking the actions of its complement as given, can deviate in a way that increases the sum of the payoffs of all of its members. Note that if a strategy is QEMSPE, then it is also QSPE, whereas the contrary is not true.

Now we just want to outline why it could be so complex (specially when n is big) the problem of checking if a coalitional simple strategy profile is QSPE. To avoid the coalitional deviations we need to punish all the coalitions but the grand one (note that it is not possible to punish the grand coalition). Even in the simplest case of a unique punishment for each coalition (irrespective of the phase in which the deviation has taken place) we have to avoid deviations of any of the $2^n - 2$ coalitions from any of the $2^n - 1$ outcome paths. Besides each coalition could deviate for only one period, for any finite number of periods, or even, forever. Furthermore, we have to take into account the *coordinated deviations* which will be explained in detail in Section 3. As we will see, potentially, these coordinated deviations could be infinitely complex.

Another important contribution of this work is to simplify considerably this problem. Proposition 1 of Section 3 generalizes a similar result of Abreu in (1989) *In this Proposition we prove that only one shot deviations have to be checked to avoid any coalitional deviation* where a one shoot deviation is a single-period deviation followed by sticking to the strategy in the sequel.

As a conclusion we summarize the main contributions of this paper.

We introduce motivatedly the Quasi Strong Perfect Equilibrium and Quasi Even More Strong Perfect Equilibrium. We introduce also the coalitional simple strategy profiles. We include an example which shows the complexity of the problem of the coordinated deviations. We prove the aforementioned Proposition 1. We use this results in the Cournot setting with usual assumptions to prove that in the Cournot supergame with any number of players it is possible to sustain the symmetric monopoly outcome by means of a QSPE strategy. Besides we show that there exists a variety of strategies that allow us to obtain the same result. We conclude with some comments on related work, with special attention to the paper of M. Horniaček (1996)

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