Forecast horizon for one-demensional control problems

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Abstract

We are concerned with an optimal control problem with time-depending parameters. Two kinds of such dynamic parameters are considered. The first is a scalar: the end of the interval $[0, T]$ on which the problem is defined. The second is a function d defined on R^+ and describes some other dynamic data of the controlled process and of the objective functional.

Very often, in practical problems, the actual dynamic parameters are not known precisely. It is obvious that T is large number but its exact value as well as the forecast of the values of the data d further into the future are rather unreliable. On the other hand, what is most important to the decision maker is a solution (optimal control) restricted to a given interval, say $[0, t']$. In some cases, it may happen that the optimal control restricted to $[0, t']$ is not affected by future data beyond a certain horizon t'', $t'' \geq t'$. If t'' is finite it is called a $forecast\ horizon$.

In the paper the forecast horizons are determined for two kind of one-dimensional control problems.

The first one is deterministic. The controlled process is given by the differential equation

$$\begin{aligned} x'(t) &= u(t) - d(t) - a\,x(t); \quad x(0) = 0, \\ \text{with the constrain} &: \quad x(t) \geq 0, \quad u(t) \geq 0 \\ \text{where} \quad a &\geq 0, \quad d(t) \geq 0. \end{aligned}$$

A convex cost functional is associated with the process $x(.)$ and control $u(.)$. A class of dynamic parameters $d(.)$ for which the forecast horizons can be explicitly obtained is determined.

The second problem is a discrete-time, stochastic, linear control problem. This time the controlled process is described by means of a sequence of Markov transition functions and two deterministic sequences. For some classes of sequences the forecast horizons are explicitly obtained.