Long-run behavior of games with many players

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Abstract: One of the fundamental problems in game theory is that of equilibrium selection in games with multiple strict Nash equilibria. One of the selection methods is to construct a dynamical system where in the long run only one equilibrium is played with a high frequency. Here we will discuss adaptive dynamics where agents adapt in some optimal way to the environment created by other players and with a small probability they make mistakes. To describe the long-run behavior of such stochastic dynamics, Foster and Young [1] introduced a concept of **stochastic stability**. A configuration of a system is stochastically stable if it has a positive probability in the stationary state of the above dynamics in the limit of no mistakes. It means that in the long run we observe it with a positive frequency.

In spatial games, agents are located on vertices of certain graphs and they interact only with their neighbors [2, 3, 4, 5]. In discrete moments of time, players adapt to their neighbors by choosing with a high probability the strategy which is the best response, i.e. the one which maximizes the sum of the payoffs obtained from individual games. With a small probability, representing the noise of the system, they make mistakes.

Now, for any arbitrarily low but fixed noise, if the number of players is big enough, the probability of any individual configuration is practically zero. It means that for a large number

of players, to observe a stochastically stable configuration we must assume that players make mistakes with extremely small probabilities. On the other hand, it may happen that in the long run, for a low but fixed noise and sufficiently big number of players, the stationary state is highly concentrated on an ensemble consisting of one Nash configuration and its small perturbations, i.e. configurations, where most players play the same strategy. We will call such configurations **ensemble stable.**

We will present examples of spatial games with three strategies where concepts of stochastic stability and ensemble stability do not coincide [6]. In particular, we constructed a game where a stochastically stable strategy is played in the long run with an arbitrarily low frequency. We discuss also an effect of adding a dominated strategy to a game with two strategies. The presence of such strategy may cause a globally risk and payoff-dominant strategy to be observed in the long run with a frequency close to zero. Three-player spatial games were discussed in [7] and some other spatial games in [8].

We will also review two models of adaptive dynamics of a darwinian type. Here we will consider two-player symmetric games with two strategies and two symmetric Nash equilibria: a payoff-dominant equilibrium and a risk-dominant one. In both dynamics, the selection part of the dynamics ensures that if the mean payoff of a given strategy at the time t is bigger than the mean payoff of the other one, then the number of individuals playing the given strategy should increase in $t+1$. In the first model, introduced by Kandori, Mailath and Rob [9], one assumes (as in the standard replicator dynamics) that individuals receive average payoffs with respect to all possible opponents - they play against the average strategy. In the second model, introduced by Robson and Vega-Redondo [10], at any moment of time, individuals play only one game with randomly chosen opponents. In both models, players may mutate with a small probability hence the population may move against a selection pressure.

It was shown that in the Kandori-Mailath-Rob model, the risk-dominant strategy is stochastically stable - if the mutation level is small enough we observe it in the long run with the frequency close to one. In the Robson-Vega-Redondo model, the payoff-dominant strategy is stochastically stable. It is one of very few models in which a payoff-dominant strategy is stochastically stable in the presence of a risk-dominant one. We will show that in the sequential dynamics in the Robson-Vega-Redondo model, for any arbitrarily low but a fixed level of mutations, if the number of players is sufficiently big, a risk-dominant strategy is played in the long run with a frequency closed to one [11]. It means that when the number of players increases, the population undergoes a transition between equilibria.

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