THE ROOT–MEAN–SQUARE EQUILIBRIUM IN GAME PROBLEM WITH VECTOR–VALUED PAYOFFS

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ABSTRACT

The N-persons noncooperative game with vector-valued payoffs $\langle N, \{ X^i \}_{i \in N}, \{f^i(x)\}_{i\in N} \rangle$ is considered. Here $N = \{1,\ldots,n\}$ is a set of players. The set $X^i \subset R^{ki}$ is composed by the strategies $x^i = (x^i_1 ,\ldots, x^i_{ki})$ of a player $i \in N$. The collection of the strategies of all players is known as a situation and set of all situations are $X = \Pi X^i$. The vector-valued functions $f^i : X \to R^{li}$ are given. For each situation function f^i puts in correspondence a payoff vector $f^i(x) = (f^i_1(x),\ldots, f^i_{li}(x))$ of the player $i \in N$.

The party of a game develops as follows: each player $i \in N$ chooses the strategy $x^i \in X^i$, so there is a situation $x \in X$. After that, the players receive payoffs $f^i(x) = (f^i_1(x),\ldots, f^i_{li}(x))$. The purpose of the player $i \in N$ is a choice of such strategy, which to achieve probably greater value of each component of the "own" vector-valued payoff function.

For this problem there is a well-known decision. It is Nash – Pareto equilibrium. But there are a lot of such solutions. The choice of the unique equilibrium is a nontrivial task. The refinement of the Nash – Pareto equilibrium is discussed. As in multi-objective problem, we are proposed the root-mean-square equilibrium. It is a strong refinement of the Nash-Pareto equilibrium, as the example is shown

<u>Definition.</u> The situation $x^\bullet = (x^{1\bullet}, \ldots , x^{n\bullet}) \in X$ is called the root – mean - square equilibrium in a game problem with vector – valued payoffs (1), if $\forall\, i \in N$, $x^i \in X^i$ is fulfilled an inequality

$$d (f^{i\bullet} (x^{-i\bullet}), f^i (x^{-i\bullet}, x^{i\bullet})) \le d (f^{i\bullet} (x^{-i\bullet}), f^i (x^{-i\bullet}, x^i)) .$$

Here the $f^{i\bullet}(x^{-i\bullet})$ is an "utopian" point.

This definition is common enough. It carries out in itself the features of a game equilibrium and vector root-mean-square solution. At first, it includes, how a special case, the definition of Nash equilibrium for game (1) under $l_i = 1$, $i \in N$. Secondly, it is reduced to a root-mean-square solution in a vector optimization problem.

<u>Statement.</u> Let's in N-persons noncooperative game with vector-valued payoffs for each players $i \in N$ the following conditions are hold:

а) the strategy set $X^i \subset R^{ki}$ is a nonempty convex compact set;

б) the vector-valued function $f^i (x)$ is continuous on X;

в) for any strategy collection $x^{-i} \in \Pi_{j\in N,\, j\neq i} X^j$ each component $f^i_j (x^{-i}, x^i), j = 1,\ldots,l_j$, of the vector-valued function $f^i (x^{-i}, x^i)$ is concave on X^i .

Then in this game there is the root-mean-square equilibrium.