

Parametric Rationing Methods

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Abstract: In a rationing problem, a single homogeneous good is allocated among agents with possibly complex characteristics, or types. When types are single positive numbers (agents' claims), Young's theorem says that in the presence of continuity, a method of rationing is consistent and symmetric if and only if it can be represented by a continuous parametric function. This theorem is generalized to all separable type spaces. Related results include a characterization of non-continuous parametric methods and a simple criterion for deciding when a two-agent method can be consistently extended to a multi-agent method. An application to the multi-category bankruptcy problem is described.

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Extended abstract: The problem of rationing is how to allocate a single homogeneous good or necessary burden among two or more agents. The good may be divisible or it may come in whole units; constraints may be imposed on the amount an agent can receive or, more generally, on the set of possible allocations. The agent's characteristics—his *type*—may include a claim to the good (represented by a positive number), a vector of claims of varying priorities, a record of need or merit, a utility function, or his most desired amount of the good.

Examples of rationing problems with complex types abound. In bankruptcy, bequest, or liquidation, an estate must be divided among debtors, heirs and/or shareholders. In elections with multiple large districts, seats must be apportioned among districts and next among parties. As a result of war-induced

shortages or in centrally-planned economies, gas, meat, even sugar and milk are commonly rationed. In privatization, shares in a property are allocated among potential buyers, employees, or all citizens. In restitution, previous (possibly many) owners of an estate are compensated. In a company, the workload may be allocated among employees who may have different preferences over their time contributions. In clubs, partnerships, and joint ventures costs and possibly profits are allocated among members and partners. The ultimate cost-sharing problem of financing a joint venture is the system of taxation for a political community.

This paper follows Young's (1987) classic study of an especially interesting class of solutions to rationing problems that can be represented with a parametric function. A rationing method is parametric if every type can be assigned a function that specifies his allotment for all possible availability levels of the good. When the agent's type is a single positive real number and when continuity of the method is assumed, Young proved that continuous parametric methods are identical with methods that satisfy simple properties of symmetry and consistency. It turns out that this characterization can be extended to virtually all imaginable rationing problems in which agents' types come from a separable topological space (such as \mathbb{R} , \mathbb{R}^k , or certain utility function and preference relation spaces, so long as an appropriate topology is assumed). The second result settles the role of continuity in the characterization. A method is parametric if and only if it is symmetric and consistent, and if there exists a separable topology on the space of types such that the method is continuous. Finally, the following question is considered: When can a rationing method defined for pairs of agents be extended to a consistent multi-agent method? A simple criterion for the existence of such a consistent extension is provided.