Perturbation Theory and Mathematical Programming Problems

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The *perturbation method* is one of the fundamental tools of Applied Mathematics. A formal application of the method assumes the following scheme. The original optimisation problem is embedded in a family of problems depending on a parameter (possibly, vector valued) that takes its values in a neighbourhood of some nominal value (usually zero). The importance of this approach stems from the fact that in a vast majority of optimisation problems arising in applications the data are not known precisely and the solutions cannot be relied upon unless they have desirable stability properties. If the solution and/or optimal value of the perturbed problem depend continuously on the parameter when it tends to zero, the problem is called *regularly perturbed*, otherwise it is called *singularly perturbed*. The *reduced* or the *unperturbed* problem is obtained by merely setting the perturbation parameter to its nominal zero value.

Both regular and singular optimisation problems are research subjects in which there is a lot of research activity internationally. For regularly perturbed problems much attention is paid to an evaluation of the rate of the convergence of the "perturbed" solution to the reduced one and also to a construction of corrections allowing a better approximation of the perturbed solution. In singularly perturbed problems the reduced problem does not provide a proper approximation for the perturbed one and hence other - more subtle - techniques are required to understand the asymptotic behaviour of solutions. One new approach is to develop a unified analysis of the regularly and singularly perturbed problems within the framework of series expansions that can be derived with the help of powerful tools of classical analysis. In this seminar an outline of this approach is presented together with a number of motivating applications.