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Risk aggregation in the Radner-type general equilibrium model

The Pareto-optimal states of an economy turn out to depend only on the aggregate risk with respect to total resources of the economy, and consequently to minimize that risk, when investors are strictly risk averse (Werner [1998]). This result has been obtained for a for-two period exchange economy under uncertainty (Werner [1985]). In this context the paper aims to generalize and enhance the above in two directions. Firstly, the exchange economy is extended to a Debreu model with the private ownership (Debreu [1959]), where, secondly, the risk aggregation is referred to the production as well as profit distribution. For this purpose, a three-period Debreu economy is constructed, where a Radner equilibrium is defined and examined.

For such states a suitable version of the first fundamental theorem of welfare economics is established. The resulting Pareto-optimal states are studied with reference to risk minimizing. This turns out to hold for a few types of the risk aggregation, when investors are risk averse. The aggregation is referred to both individual and total production as well as profit distribution.

Thus, the paper is divided into five parts. In the next section, the model is constructed, where the concept of a Radner equilibrium is defined. Section 3 is focused on the problem of the existence of a Radner equilibrium. The respective theorem is proved. Section 4 deals with a production-social preference relation as well as the Pareto-optimal properties of Radner equilibria. Finally, in Section 5 the risk aggregation, and consequently its minimizing in such states is examined.

In the research on the models of multi–period economic activity, the analysis of attitudes of market participants under risk leads to risk aggregation and, in consequence, to its minimizing in optimal state of economy. In the context of the obtained results (Werner [1998]) the problem of risk aggregation with respect to other parameters are characteristic for agents' activity can be raised. Except for the total resources of economy there are: individual and total production, profit value to allocate to dividend and the volume of securities. Then, the analysis of individuals' (producers' as well) attitudes towards risk is justified, as well as their connection with aggregate risk types to risk minimizing.

There are two lines of reasoning developed under risk. They enable us, on one hand, to define precisely producers' attitude towards risk, and, on the other hand, Pareto optimality of an economy.

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Except for the social preference relation, which is based on consumers' individual preferences, we suggest a type of the social preference which takes into account producers' preferences depending on the price system *p*

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The total endowments ϖ^{1s} are aggregated in the meaning that they may be the same in some states, even if some consumers' endowments are not equal in these states. For this reason, they do not reflect the whole risk in an economy at the date 1. The risk connected with total endowments is such that the consumers have to meet it. We can reduce it to minimum in an attainable state of economy in which consumer's plans x_i, $i \in I$, depend only on aggregate risk. This property of minimal risk the optimal states have, has been proved for two-dates model of economy without production (Werner [1985]). In our model of economy with production, we examine producers' activity as a risk source, as well as the total endowments, both of which can reduce the aggregate risk. So, we assume the following definition.