Competition, Cooperation, and Authorization

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Abstract. Multi-agent systems considered in the paper consist of a finite number of agents, positions of which can be changed by system actions, and of an evaluation function which assigns to each agent a value of its current position (as e.g. the distance from the intended target). The set of all possible values is ordered; the intention of each agent is to reach a position with the minimum value. Any system action can decrease position values of some agents (the winners) and increase those of the others (the losers); consequently, an action execution can create conflicts among its participants (winners and losers); arbitrary resolutions of such conflicts can prevent some agents of reaching their goals. The present paper is aiming to formulate conditions that must be fulfilled by actions to guarantee each agent reaching its intended final situation.

Keywords: multi-agent system; conflicts; cooperation; concurrency; nondeterminism; distributed algorithms.

> *For whosoever hath, to him shall be given, and he shall have more abundance; but whosoever hath not, from him shall be taken away even that he hath.* [**Mat 13:12**]

1 Introduction.

One of characteristic features of multi-agent systems is a competition between agents or groups of agents. Agents may compete for resources, for access to devices, for authorization to undertake some actions, for other privileges that can be granted or refused. On the other hand, agents can cooperate, if such a cooperation can be favorable for them: some privileges can be offered only for sufficiently great number of applicants, or some resources can be allocated for a group of agents with a common purpose. Competition or cooperation among agents can vary during the system activity, and then instantaneous coalitions among agents have a dynamic character.

Agents are acting to reach their goals. To do it, they act in a rational way, evaluating their situations at any moment of the system run and try to improve

⋆ partially supported by KBN grant nr 7 T11C 006 20 and EC Framework Program 5 IST-1999-10298 "ALFEBIITE"

their situation by performing a suitable action. However, an action being favorable for some agents may be unfavorable for others; the competition between agents results in choosing an action improving the situation of some agents, but at the same time worsening it for others. In effect of such a competition none of agents may reach its target: making a progress in one step, an agent can lose its gain in the next one. There is a simple strategy to avoid infinite and useless competing: an agent being in the best situation should have a chance to improve it, at the cost of others. This strategy has been already applied in some routing algorithms, e.g. in [2] and message distribution [4]. This paper is aiming to formulate precisely this strategy in an abstract framework, neglecting non-substantial features of multi-agent systems but retaining those enabling proper formulation of the required properties.

To grasp the essential phenomena of such a strategy, the presented set-up of the multi-agent systems is limited to the basic system components: *agents*, *states* of the system that can be transformed in effect of the system *actions*, and *evaluation* of situation of any agent at an instantaneous system state. Value of an agent situation is a measure of how far from the target the agent is.

The order of action execution as well as the choice of action is not specified; some actions with disjoint sets of participants can be executed independently of each other. Systems discussed here are uniform (no particular agent is distinguished), local (any change of state concerns only participants of the changing action, leaving positions of other agents unchanged), self-stabilizing (it can start with an arbitrary initial state), non-deterministic (there is no prescribed order in which actions are executed), and concurrent (some actions can be performed independently of each other).

In the paper the standard mathematical notions are used, with $\mathbf{N}$ denoting the set of all non-negative integers ordered in standard way. First, the description of the structure and behavior of the discussed systems is given, and next some limitations of its behavior that guarantee all agents to reach eventually their goals. Finally, some extensions and consequences of the approach is given.

2 Abstract multi-agent systems.

An abstract multi-agent system considered here and called *ams* for short consists of a finite set of agents, a set of system states, a set of actions, and an evaluation function, which to each agent g in any state s assigns a current value $V(g,s)$ of its situation in this state; this value is called here the *position value* of agent g at state s. The set of all position values is ordered making possible their comparison. Formally, system $\mathbf{A}$ is defined as a tuple

$$\mathbf{A} = (G, S, R, V)$$

where

G is a finite set (of *agents*),
S is a set (of *states*),

$$R \subseteq S \times S \text{ is a set (of } \textit{actions}\text{)},$$
$$V : G \times S \longrightarrow D, (\textit{valuation} \text{ function}),$$

where D is a set (of *agent position values*). Sets G, S, D are assumed to be disjoint and non-empty. Instead of $(s', s'') \in R$ write $s' \rightarrow_R s''$, or simply $s' \rightarrow s''$ if R is understood; similarly, instead $(s', s'') \notin R$ write $(s' \not\rightarrow_R s'')$ (or $(s' \not\rightarrow s'')$, resp.). Set D is totally ordered by $<$ relation; as usual, $d' \leq d''$ means $d' < d''$ or $d' = d''$; $d' > d''$ means $d'' < d'$. Set D is assumed to meet the "minimum property" guaranteeing that each non-empty subset of D has the least element. The least element of the whole set D is denoted by $\mathbf{0}$. Value $V(g, s)$ will be called the *position value* of g at state s and is intended to estimate how far agent g at state s is from its goal. If $V(g, s) = \mathbf{0}$, state s is the target state for agent g. Let $r = (s' \rightarrow s'')$ be an action. Agent g is a *winner* of r, if $V(g, s') > V(g, s'')$, and is a *loser* of r, if $V(g, s') < V(g, s'')$; winners and losers in r are *participants* of r; remaining agents of the system are said to not *participate* in r. In other words, g is a winner (loser) of an action, if in effect of this action the distance from its target decreases (increases, resp.). Agent g is *dismissed* in state s, if $V(g, s) = \mathbf{0}$, and *active*, otherwise. A state is *active*, if there is at least one agent active in it, and *terminal*, otherwise. The value $V(g, s)$ will be also called the *position* or the *situation* of agent g at state s.

The intuition behind the above definition is the following. There is a group of agents that intend to reach their individual goals executing some actions. Each action concerns only its participants, i.e. some agents from the group. Actions cause change of the situation of their participants, and leave situation of remaining agents unchanged; thus, actions are local changes of agents situations. The situation of an agent is measured by its distance from its targets; distances are represented by elements of an ordered set: for any agent, the smaller is the distance from the target, the better is its situation.

System axioms. The following axioms concerning agents and actions are assumed to hold for all states s', s'' and agents g of the system:

(A1) $s' \rightarrow s'' \Rightarrow \exists g : V(g, s') > V(g, s'')$
(there is a winner of any action);

(A2) $V(g, s) > \mathbf{0} \Rightarrow \exists s' : s \rightarrow s' \wedge V(g, s) > V(g, s')$
(any active agent at any state has a chance to win);

(A3) $V(g, s) = \mathbf{0} \Rightarrow \forall s' : s \rightarrow s' \Rightarrow V(g, s') = \mathbf{0}$
(dismissal is permanent).

In other words, for any active agent in any situation there exists at least one action favorable for this agent, and the other way round: in any action there is at least one winning agent. Below we give an example of an *ams* system.

Competition and cooperation. Depending on possible actions, agents can cooperate, compete, or be independent at some states. Agents winning an action can be viewed as cooperating, while all losers as unsuccessful competitors against the formers. The choice between two actions that are favorable for some agents causes no conflict between them; a conflict arises between winners and losers, since any loser could be a winner with another choice of action. A conflict can also arise between two agents that can win two separate actions, if a result of one action leads to a state from which the second agent has less favorable possibilities of acting. Two actions are independent, if they do not interfere with each other. Independent actions have disjoint sets of participants, executed in any sequence give the same result, making possible to treat them as a single action, with the union of participants. If two actions are independent, execution of one of them does not prevent the other from being executed. Two agents can be partners, if both of them participate in an action; allies, if both of them are winners of an action, or competitors, if one of them is a winner and the other a loser in an action. The choice of an action, which implies the choice of winning and losing agents, is non-deterministic, unless some rules of priority, called here 'authorization rules', are introduced to the agent's community. Two such rules will be formulated in the sequel.

Example 1. Let $\mathbf{E} = (G, S, R, V)$ be an *ams* system with

$$G = \{g_1, g_2\},$$
$$S = \{s_0, s_1, s_2, s_3, s_4, s_5\}$$

and with R and V given by the following tables:

Actions:

R	s_1	s_2	s_3	s_4	s_5
s_0	$\to$	$\to$	$\times$	$\times$	$\times$
s_1	$\times$	$\to$	$\to$	$\times$	$\times$
s_2	$\to$	$\times$	$\times$	$\to$	$\times$
s_3	$\times$	$\times$	$\times$	$\times$	$\to$
s_4	$\times$	$\times$	$\times$	$\times$	$\to$

Valuation:

V	g_1	g_2
s_0	3	3
s_1	2	3
s_2	3	2
s_3	1	4
s_4	4	1
s_5	0	0

States will be represented by the corresponding valuations of agents; that is, state s_0 is represented by (3,3), state s_1 by (2,3), ..., state s_5 by (0,0). In the first table sign $\to$ (sign $\times$) in column s' and row s'' denotes $s' \to_R s''$ ($s' \not\to_R s''$, resp.); values $\{0, 1, 2, 3, 4\}$ of function V are supposed to be ordered in the natural way. It is easy to check that the above definition meets axioms $A1, A2, A3$. The winner of action $(3, 3) \to (2, 3)$ is g_1; the winner of action $(3, 3) \to (3, 2)$ is g_2; agent g_2 does not participate in action $(3, 3) \to (2, 3)$, agent g_1 does not participate in action $(3, 3) \to (3, 2)$. Both action have only one participant each, g_1 and g_2 neither cooperate, nor compete. However, actions $(3, 3) \to (2, 3)$ and

$(3,3) \to (3,2)$ are not independent since they cannot be executed simultaneously - there is no state (2,2) which could be the effect of such an execution. Agent g_2 is a loser in $(2,3) \to (1,4)$ and agent g_1 is a loser in $(3,2) \to (4,1)$. In actions transforming situation $(2,3)$ both agents compete; if the result of action is $(1,4)$, agent g_1 wins and agent g_2 loses; if the result is $(3,2)$, g_2 wins, and g_1 loses. In actions transforming states (1,4) and (4,1) both agents cooperate, win, and become dismissed. This system can be visualized by the picture presented in Fig.1. □

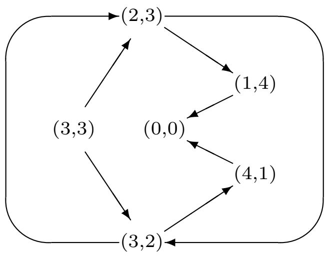

Fig. 1. The diagram of system **E**

3 System runs.

Let $\mathbf{A} = (G, S, R, V)$ be an arbitrary agent system, fixed from now on. Call state s *live*, if there exists another state s' with $s \to s'$; denote the set of all live states of $\mathbf{A}$ by $\mathbf{L}$. Any sequence in $\mathbf{N} \longrightarrow S$ is a *run* of the system, if for all $i \geq 0$

$$s_i \in \mathbf{L} \wedge s_i \to s_{i+1} \vee s_i \notin \mathbf{L} \wedge s_i = s_{i+1}. \tag{1}$$

Run (1) is said to *terminate*, and state s to be its *last* state, if there is $k \geq 0$ such that $s = s_k \notin \mathbf{L}$. A run is *successful*, if it terminates and all agents are dismissed at its last state: $V(g, s) = \mathbf{0}$ for all $g \in G$ and for the last state s of the run in question.

Proposition 1. *Any run of ams is either successful, or non-terminating.*

Proof. In the opposite case there would be a terminating run with some agents active at its last state; then the last state would be live (by Axiom A1), contradicting the definition of runs. □

Example 2. The sequence

$$(3,3) \to (3,2) \to (2,3) \to \cdots \to (3,2) \to (2,3) \to \cdots$$

is a non-terminating (hence not successful) run of the system defined in Example 1. In this run the winners alternate and no agent reaches its target (the corresponding distances are always greater than 0). However, allowing either of the two win twice gives a successful terminating run, e.g.

$$(3,3) \to (3,2) \to \cdots \to (2,3) \to (1,4) \to (0,0)$$

or, respectively,

$$(3,3) \to (2,3) \to \cdots \to (3,2) \to (4,1) \to (0,0)$$

In the above cases both agents reach their destinations (the infinite sequence of $(0,0)$ following the last state of both runs is omitted). $\square$

4 Authorizations.

Authorization is a restriction imposing some conditions on executability of system actions. Instruction $s' \to s''$ is *authorized* by condition C, if $s' \to s'' \Rightarrow C(s', s'')$; a run is authorized, if any pair of its consecutive states is authorized, and the system is authorized, if any run of it is authorized. Authorizations are introduced to prevent systems from unsuccessful runs, either nonterminating (livelocks), or terminating (deadlocks). The most liberal authorization is given by the condition identically true **T**; according to this authorization all possible runs of the system are authorized. As it is seen from the Example 2 some of runs of such systems can be non-terminating. The most restrictive authorization is given by the condition identically false **F**, i.e. by the never fulfilled condition; in this case no action is authorized and each run terminates at its first state. Clearly, all runs of such systems are terminating, but not necessarily successful. We are seeking for something in between these two extremities. In what follows we shall discuss two 'intermediate' (non-trivial) authorization conditions that guarantee all runs successful and do not admit deadlocks; moreover, they are local, i.e they refer exclusively to states of participants of the authorized action. Authorized actions do respect axioms A1 and A3, but they do not observe axiom A2 any more: not all agents active in a state can be winners.

Authorization W. The first authorization, called here **W**, is given by the following restrictions:

$$\begin{aligned} s' \to s'' \Rightarrow \exists g_0 : V(g_0, s') > V(g_0, s'') \ \wedge \\ (\forall g : V(g, s') < V(g, s'') \Rightarrow V(g_0, s') \le V(g, s')) \end{aligned} \tag{2}$$

(there exists a winner in a position not worse than any loser). Intuitively, an action is authorized by **W**, if at least one of its winners is in better or at least equal position than any of its losers. Observe that the above condition is local. We shall prove that authorization **W** prevent the system from non-terminating runs, which is the main objective of introducing authorization conditions. For any live situation (with at least one active action) there exists also at least one action enabled and authorized by $\mathbf{W}$; it guarantees that the authorization does not cause the system deadlock, i.e. that any terminated and authorized run is successful. We prove in the sequel that any authorized run terminates.

Proposition 1 *Authorization (2) is equivalent to the following:*

$$s' \to s'' \Rightarrow \exists g_0 : V(g_0, s') > V(g_0, s'') \ \wedge \\ (\forall g : V(g, s') \neq V(g, s'') \Rightarrow V(g_0, s') \leq V(g, s')) \tag{3}$$

(a participant in the best position wins).

Proof. Let (2) holds and $s' \to s''$. Then there exists a winner in a position not worse than any loser. Consequently, the participant of $s' \to s''$ in the best position is also a winner, which implies (3). Conversely, if (3) holds, then clearly there exists a winner in a position not worse than any loser. It proves the proposition. □

The difference between conditions (2) and (3) is that the winner of any action authorized by (3) has the best position among *all* its participants, while in actions authorized by (2) it has a position not worse than any of losers only. In view of Proposition 1 both formulations are equivalent.

Example 3. Sequences

$$(3, 3) \to (3, 2) \to (4, 1) \to (0, 0),$$

and

$$(3, 3) \to (2, 3) \to (1, 4) \to (0, 0)$$

are successful runs of the system defined above in Example 1; they are the *only* runs starting with $(3, 3)$. Action $(3, 2) \to (4, 1)$ is the only authorized action at state $(3, 2)$, and action $(2, 3) \to (1, 4)$ is the only authorized action at state $(2, 3)$. The system $\mathbf{E}$ defined in Example 1 and authorized by $\mathbf{W}$ is presented in Fig.2. □

Proposition 2 *Any terminating run of any system authorized by* $\mathbf{W}$ *is successful.*

Proof. If there exists any active agent at a state, then there exists an authorized action at this state, namely the action improving position of the best situated active agent. Therefore a state with a number of active agents cannot be the last one in a run. □

Theorem 1 *Any run of any system authorized by* $\mathbf{W}$ *is successful.*

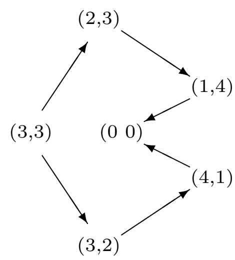

Fig. 2. System **E** authorized by **W**

Proof. Let $\mathbf{A} = (G, S, R, V)$ be a multi-agent action system with $|G| = N$ and let $s \in \mathbf{N} \longrightarrow S$ be a run of $\mathbf{A}$ authorized by $\mathbf{W}$ with $r_i = (s_i, s_{i+1})$ for each $i \geq 0$. Define agents $g_1, g_2, \ldots, g_N$ and integers $i_1, i_2, \ldots, i_N$ as follows. Set $M_1 = \min\{V(g, s_i) \mid g \in G, i \geq 0\}$ and let agent g_1 and integer i_1 be such that $V(g_1, s_{i_1}) = M_1$. Suppose g_k, i_k have been already defined; then define $M_{k+1} = \min\{V(g, s_i) \mid g \in G - \{g_1, \ldots, g_k\}, i \geq i_k\}$ and let agent g_{k+1} and integer i_{k+1} be such that $i_{k+1} \geq i_k$ and $V(g_{k+1}, s_{i_{k+1}}) = M_{k+1}$. Since the total number of agents of the system is equal to $N, \{g_1, g_2, \ldots, g_N\} = G$. Clearly, $i_1 \leq i_2 \leq \cdots \leq i_N$. Observe that values M_i exist due to the minimum property of set D.

Now, prove that $V(g_1, s_i) = M_1$ for all $i \geq i_1$. Indeed, by minimality of M_1, there cannot be any state s_i of the run with $i \geq i_1$ and with $V(g_1, s_i) < M_1$. Consequently, due to authorization condition (2), g_1 cannot be a winner in any action r_i with $i \geq i_1$, since otherwise g_1 should improve its own situation yielding $V(g_1, s_i) < M_1$. But also g_1 cannot be a loser of any action r_i with $i \geq i_1$, since then, by axiom A1 and the authorization condition (2), there would be a winner in r_i, say h_i, with $V(h_i, s_i) \leq M_1$, and in the effect of this action r_i it would be $V(h_i, s_{i+1}) < M_1$, again contradicting the definition of M_1. Therefore, $V(g_1, s_i) = M_1$ holds for all states s_i with $i \geq i_1$ proving g_1 not participating in any action r_i with $i \geq i_1$.

Suppose we have already proved that none of agents $g_1, g_2, \ldots, g_k$ participate in any action r_i with $i \geq i_k$ and prove $g_1, g_2, \ldots, g_k, g_{k+1}$ do not participate in any action r_i for $i \geq i_{k+1}$ either. Indeed, since $g_1, g_2, \ldots, g_k$ do not participate in any actions r_i with $i \geq i_k$, none of them can be a winner or a loser in r_i with $i \geq i_k$. By definition of i_{k+1}, M_{k+1}, we have $V(g_{k+1}, s_{i_{k+1}}) = M_{k+1}$; clearly, g_{k+1} cannot be a winner of actions r_i with $i \geq i_{k+1}$, contradicting the definition of since otherwise it would be $V(g_{k+1}, s_i) < M_{k+1}$ for some $i \geq i_{k+1}$, M_{k+1}. But g_{k+1} cannot be a loser of actions r_i with $i \geq i_{k+1}$ as well, since otherwise an agent, say h_i, out of $g_{k+2}, g_{k+3}, \ldots, g_N$ would be a winner with $V(h_i, s_i) \leq M_{k+1}$, and with $V(h_i, s_{i+1}) < M_{k+1}$, contradicting the definition of M_{k+1}. Thus, $V(g_{k+1}, s_i) = M_{k+1}$ for all $i \geq i_{k+1}$. Repeating similar arguments and taking into account inequalities $i_1 \leq i_2 \leq \cdots \leq i_N$ we prove that all agents of the system are independent of any action r_i with $i \geq i_N$. From this it follows that the length of the run is not greater than i_N, proving the run to terminate. By Proposition 2, all agents of the system are dismissed at the last state of the run. □

Authorization S. To define this authorization, let the set of all agents be linearly ordered by $\prec$ relation. This ordering may be arbitrary, without distinguishing any agent; it can be established e.g. by a distributed enumeration procedure (see [3]), if such a procedure exists, not assuming any predefined hierarchy of agents (preserving uniformity of the agent's society). We shall refer to relation $\prec$ as to the "older" relation: g_1 is older that g_2 means $g_1 \prec g_2$. Then the authorization condition **S** is defined as follows:

$$s' \to s'' \Rightarrow \exists g_0 : V(g_0, s') > V(g_0, s'') \ \wedge \\ (\forall g : V(g, s') < V(g, s'') \Rightarrow g_0 \prec g) \tag{4}$$

(there exists a winner older than any loser). Similarly to authorization **W** we have the following property:

Proposition 3 *Authorization* **S** *is equivalent to the following:*

$$s' \to s'' \Rightarrow \exists g_0 : V(g_0, s') > V(g_0, s'') \ \wedge \\ (\forall g : V(g, s') \neq V(g, s'') \Rightarrow g_0 \prec g) \tag{5}$$

(The oldest participant wins).

Proof. Similar to that of Proposition 1. □

Observe that the above authorization condition, similarly to the restriction given by **W**, is local; "the oldest participant" mentioned here does not mean "the oldest agent" in the system; there can be other older agents in the system not participating in the authorized action.

Example 4. Suppose in Example 1 agent g_1 is older that agent g_2, i.e. $g_1 \prec g_2$. Then (s_0, s_1, s_3) is the only run starting with s_0 of the system **A** authorized by **B**. If it were $g_2 \prec g_1$, the only such run would be (s_0, s_2, s_3). □

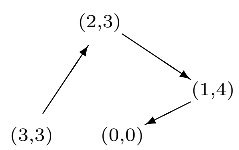

Fig. 3. System **E** authorized by **S** with $g_1 \prec g_2$

Theorem 2 *Any complete run authorized by* **S** *is successful.*

Proof. The proof is also similar to that of Theorem 1. Let, as above, $\mathbf{A} = (G, S, R, V)$ be a system with $G = \{g_1, g_2, \ldots, g_N\}$, and let $g_i \prec g_j \Leftrightarrow i < j$. Let $s \in \mathbf{N} \longrightarrow S$ be a run of $\mathbf{A}$ authorized by $\mathbf{S}$ with $r_i = (s_i, s_{i+1})$ for each $i \geq 0$. If run s is terminating, then at its last state all agents must be dismissed, since otherwise any action favorable for the oldest active agent would be possible, contradicting the definition of the run. Suppose the run s is not terminating. Since the oldest agent g_1 cannot be a loser, it is either a winner or not participating in all actions of the run. Therefore, its position value cannot increase; due to the minimum property this value cannot decrease infinite number of times, hence there is $i_1 \geq 0$ such that the oldest agent does not change its value in all actions r_j with $j \geq i_1$. That is, g_1 is not participating in any action r_i for $i \geq i_1$. Suppose $g_1, g_2, \ldots, g_k$ are not participating in all actions r_i with $i \geq i_k$ for some i_k; then g_{k+1} cannot be a loser in any action r_i with $i \geq i_k$; since g_{k+1} can be a winner only finite numbers of times in sequence $(s_{k+1}, s_{k+2}, \ldots, s_{k+n}, \ldots)$, there exists $i_{k+1} \geq i_k$ such that $g_1, g_2, \ldots, g_k, g_{k+1}$ are not participating in any action r_i with $i \geq i_{k+1}$. By induction, none of actions in G are participating in any action r_i with $i \geq i_N$. It means, by its very definition, that the run is finite, a contradiction. □

Catastrophes. Sometimes it may be useful to relax Axiom A1, saying that in any action there is a winner. Let $\mathbf{A} = (S, R, G, V)$ be a system (authorized or not), in which Axiom A1 is cut out. Call action $s' \rightarrow s''$ a *catastrophe*, if for all $g \in G$ the inequality $V(g, s') \leq V(g, s'')$ holds. In other words, catastrophes are actions without winners. However, we have the following proposition.

Proposition 4 *A finite number of catastrophes occurring in runs does not prevent systems from successful termination.*

Proof. As it is seen from the system axioms for any system and any state of it there exists a successful run starting from this state. If in a run several finite number of catastrophes occur, there is a successful run starting from the state resulting from the last catastrophe. □

5 Complexity.

Let $\mathbf{A} = (S, R, G, V)$ be a system authorized by $\mathbf{W}$ with N agents and with $V \in G \times S \longrightarrow \mathbf{N}$. Define the *maximum loss* in the system as the integer

$$L = \max\{V(g, s'') - V(g, s') \mid g \in G, s' \rightarrow s'' \in R\}.$$

For any run $s = (s_0, s_1, \ldots)$ of $\mathbf{A}$ let the *initial distance* of s be the integer $M = \max\{V(g, s_0) \mid g \in G\}$.

Theorem 3 *For any run of system authorized by* $\mathbf{W}$ *with* N *agents, initial distance* M*, and maximum loss* L*, the number of steps is bounded by integer* S*, where*

$$S = M \cdot \sum_{i=0}^{N-1} (L+1)^i. \tag{6}$$

Proof. Denote by $S(n,m)$ the upper bound of number of steps needed for reaching by n agents the terminal state of the run while starting from positions values not greater than m and assuming the maximum loss not greater than L. Since the system is authorized by $\mathbf{W}$, agents in their best positions (with the smallest position values) cannot lose; in effect, dismissing one of them requires at most m steps in which the best situated agents win. In particular, a system with only one agent and the initial distance m needs m steps to reach its terminal state. Since a success of one agent can create failures for the others, any win of an agent in its best position can increase by L position values of all remaining agents. Due to commutativity, the order of increasing and decreasing operations does not influence on the total number of system steps; hence, agents that win m times can be viewed as increasing initial distance of losing agents by $m \cdot L$. It leads to the following recursive dependency for all $n \geq 1$:

$$\begin{aligned} &S(1,m) = m, \\ &S(n,m) = m + S(n-1, m + m \cdot L), \qquad (n > 1) \end{aligned}$$

with the following explicit form:

$$S(n,m) = m \cdot \sum_{i=0}^{n-1} (L+1)^i.$$

It completes the proof by setting $S = S(N,M)$. □

Observe that in the above proof the most pessimistic variant of a run has been discussed, namely when a win of one agent causes the maximum lost for all agents in worse (or equal) position.

In Theorem 3 nothing has been said about the number of steps needed to dismiss a concrete agent, since it is not known *a priori* which agent reach the best (or worst) position in the course of a system action. In case of systems authorized by condition $\mathbf{S}$ it is not so, as it is shown by the following theorem. Let the set of agents $G = \{g_1, g_2, \ldots, g_N\}$ be ordered by $\prec$ relation in such a way that $g_1 \prec g_2 \prec \ldots \prec g_N$ and let $\mathbf{A} = (S, R, G, V)$ be a system authorized by $\mathbf{S}$ with $V \in G \times S \longrightarrow \mathbf{N}$.

Theorem 4 *The number of steps needed to dismiss agent* g_i *in a run of system authorized by* $\mathbf{S}$ *is bounded by* S_i*, where*

$$S_i = M \cdot (L+1)^i, \tag{7}$$

where M *is the maximum initial distance of the run, and* L *is the maximum loss in a single action.*

Proof. The proof is similar to that of Theorem 3, but somewhat simpler. Let $S_i, i = 1, 2, \ldots, N$, be a bound for the number of steps agent g_i needs to reach its target. Clearly, $S_1 = M$, since agent g_1, according to authorization **S**, cannot lose and its distance from the goal is M. Since each win of g_1 can increase the number of steps needed for agents $g_2, g_3, \ldots, g_N$ to achieve their goals by $M \cdot L$ each, and no win of agents $g_3, g_4, \ldots, g_N$ can do the same w.r.to g_2, $S_2 = M \cdot (L + 1)$. Repeating this reasoning for the successive agents we conclude that $S_i = M(1 + L)^i$ for all $i \geq 1$. □

6 Conclusions.

A behavior of a finite set of agents capable to perform some (local) actions has been described. The purpose of any agent is to reach its goal; however, actions of agents may interfere with each other and an action favorable for some agents can turn out unfavorable for the others. The general question discussed in the paper is to impose some conditions on the behavior of agents that guarantee all agents reaching their targets. Any action changing situation of agents is local: it concerns only some agents, called the participants of this action. Each participant of an action is winning or losing: it wins, if the distance from its goal is decreasing, and losing if it is increasing in effect of this action. The choice of an action is non-deterministic; it can be viewed as an effect of resolution of the conflict between competing agents. Agents are competing, if at the same situation there are two actions, the first favorable for one agent and unfavorable for the other, and the second the other way round. Two agents can be viewed as cooperating in an action, if both of them are winning in this action. Observe that words "competing - cooperating" used here are related to a single action; in two different actions even their winners can be considered as competing, if they gain different values. The main objective of the paper is to formulate some conditions concerning action execution that guarantee each agent to reach its goal. Among all possible actions only those satisfying so-called authorization condition can be executed at a system state. Two such authorization conditions have been formulated and proved to meet the required properties. The following features of agent systems (either following directly from definitions or very easy to be justified) have been covered in the paper:

- locality (state changes depend on and concern participants only)
- non-determinism (there is no prescribed order of action execution)
- self-stabilization (any authorized run starting with an arbitrary initial state is successful, see [1])
- concurrency (independent actions can be executed concurrently)
- uniformity (no particular agent is distinguished, in case of **W** authorization)

The intention was to keep the described system as simple as possible, limiting introduced notions just to enable formulation and proving the basic properties. In consequence:

- Goals of agents are not specified, and in consequence, not differentiated;

- Agents have no memory i.e. they are unable to retain and use information about past experiences;
- Agents have no knowledge about the world around; they know only their own and their partners relative positions with respect to their targets.
- Agents are not intelligent; in particular, they cannot predict future consequences of their moves (they cannot look ahead more than one step forward).

It is worthwhile to make here a word of comment. In any system action there are winners which can be viewed as a "force of cooperating agents" causing the action execution. Self-stabilization property within the agent system context offers a possibility to take also into account a "force of nature", which can disturb the situation of all agents, not necessarily in a favorable manner. If such a force changes the system state in a wanton way, with worsening position values of all agents, then it cannot be viewed as a system action containing always a winner. Self-stabilization property makes possible to admit a finite number such unfavorable changes ("catastrophes"), by treating them as unexpected perturbations not preventing the system from eventual completing its task.

As possible extensions of the formalism presented in the paper one could specify the following issues:

- Partial order of position values (there can be several features with non-comparable advantages that can influence on a decision of an agent),
- Relaxing Axiom A3 by introducing reactivity (new obligation rather than dismissal); it seems possible to introduce this feature in relation to authorization **S**),
- Relative position evaluation (introducing position evaluation for collectives of agents rather than for individuals),
- Coalition authorization (introducing authorization condition concerning not single agents, but their coalitions),
- Priority negotiations (by introducing more sophisticated agents' architecture that make possible some negotiation actions).

Clearly, this list does not exhaust all possible extensions of the formalism.

Acknowledgements. Corrections, comments, and remarks of an anonymous reviewer that enable to improve the paper considerably are highly appreciated.

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