Revealed preference and consistency conditions for fuzzy choice functions

Irina Georgescu
Turku Centre for Computer Science,
Institute for Advanced Management Systems Research,
Turku, Finland
Irina.Georgescu@abo.fi

Abstract

The revealed preference is a central subject in classical consumer theory. Authors like Samuelson, Arrow, Richter, Sen, Uzawa and others have proposed an axiomatic setting of revealed preference theory.

Consequently revealed preference axioms WARP and SARP and congruence axioms WCA and SCA have been considered.

An important theorem of Arrow-Sen establishes the equivalence between these axioms provided the family of budgets includes all non-empty finite sets of alternatives.

Fuzzy choice functions is a topic that appears in a lot of papers. Particularly, Banerjee studies in fuzzy context axioms of revealed preference and congruence extending some results of Arrow and Sen.

In this paper we modify the Banerjee definition of a fuzzy choice function and we study fuzzy versions of the axioms of revealed preference and congruence. Banerjee fuzzifies only the range of a consumer; we use a fuzzification of both the domain and the range of a consumer. The axioms WAFRP, SAFRP, WFCA, SFCA generalize to fuzzy choice functions the well-known axioms WARP, SARP, WCA, SCA.

Our first main result establishes some connections between WAFRP, SAFRP, WFCA, SFCA extending a significant part of Arrow-Sen theorem.

Next we introduce consistency conditions $F\alpha$, $F\beta$, $F\delta$ as fuzzy forms of Sen’s properties $\alpha$, $\beta$ and $\delta$. One first result shows that a fuzzy choice function satisfies $F\alpha$, $F\beta$ if and only if the congruence axiom WFCA holds. The second result shows that if $h$ is a normal
fuzzy choice function, then \( F\delta \) holds if and only if the associated preference relation \( R \) is quasi-transitive.